Non-linear computation of the gravity field of an aspherical planet

Virginie Belléguic, Philippe Lognonné, Mark Wieczorek, Département de géophysique spatiale et planétaire, IPGP

We present a method to compute the exact gravity field inside and outside a planet with an arbitrary density structure. The computations are done non-linearly and are therefore applicable for large asphericities, such as those on Mars, the Earth or the Moon. This method allows us to take into account all kinds of inhomogeneities (i.e lateral density variations, variations of the depth of each interface) and allows the modeling of gravity fields with lateral variations as large as about degree 200.
The lithospheric deflection associated with surface loads can be obtained from the value of the potential on each density interface and can provide an estimation of the elastic thickness of the lithosphere. These estimates can be used to place constrains on the heat flow of the planet and hence, on its thermal evolution.

Input:
- Surface topography
- Relief along any number of density interfaces
- Lateral variations of density

Output:
- g(r,θ,φ) and U (r,θ,φ) *at all points inside and on the* *surface* of the planet.

2 steps in the computation :
- calculation of U and g at an altitude R0 above the mean planetary radius (analogous to *Wieczorek and Phillips* 1998*)*
- downward propagation of U and g to each density interface

→ U and g are exactly determined on the surface **as well as on any density interface within the planet.**

Principles of the method :

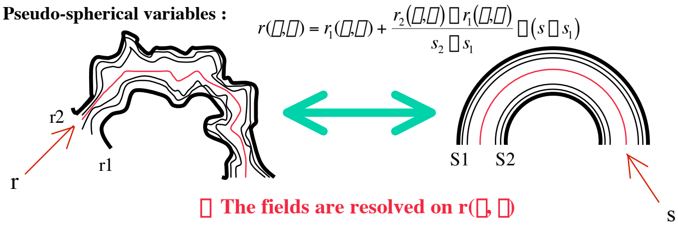
Spectral/space domain : computations are performed both in the space and spectral domain

spectral domain ⟵ Legendre Transform^{-1} / Legendre Transform ⟶ **space domain**

- All lateral variation effects are computed in the space domain (collocation grid).
- All angular derivatives are done in the spectral domain
  - Angular derivatives become scalar multiplications
  - No need to use Clebsh-Gordon coefficients
  - Reduction of the amount of computing by l^2
  - Can be applied for variations of high angular degree ($l \geq 200$)

Pseudo-spherical variables :

$$r(\theta,\phi) = r_1(\theta,\phi) + \frac{r_2(\theta,\phi) - r_1(\theta,\phi)}{s_2 - s_1}(s - s_1)$$



⇒ The fields are resolved on r(θ, φ)

- Irregular surfaces are mapped to spherical surfaces using a parameter *s*.
- Gravity and potential integrations are performed between boundaries independent of θ and φ.
- The interface effects can be considered as perturbations of the gravity equation in the mapped coordinates.

First step :

U and g are determined at R0 by the following equations

$$U(R_0,\theta,\phi) = \sum_{\ell m} B_{\ell m} Y_{\ell m}(\theta,\phi)dV'$$

with
$$B_{\ell m} = \left(\frac{1}{R_0}\right)^{\ell+1} \frac{1}{2\ell+1} A_{\ell m}$$

LT^{-1}

$$A_{\ell m} = \int_V \rho G(r',\theta',\phi')Y_{\ell m}(\theta',\phi')r'^{\ell}\, dV'$$

And,

$$g = -\nabla U$$

Downward propagation :

- Separation of the spherical and perturbation components

$$U(s,\theta,\phi) = U^0(s) + \varepsilon U^1(s,\theta,\phi)$$
$$g(s,\theta,\phi) = g^0(s) + \varepsilon g^1(s,\theta,\phi)$$

$\Delta g^1, \Delta U^1 << g^0, U^0$
- Reduction of numerical errors.
-Allows the development of the field utilizing spherical basis functions.

- U, g and their derivatives are computed on each interface (in s, θ, φ coordinates) by solving the set of first-order differential equations with either the Euler or Runge -Kutta method :

$$g = \nabla U$$
$$\nabla . g = -4\pi\rho G$$

This computational method is a part of a global project studying the lithospheric structure and composition of martian volcanoes.
Current crustal thickness models of Mars are computed under the assumption that the crust is homogenous in composition and hence density. This assumption is likely incorrect for the martian volcanoes and Tharsis rise.
A goal of this project is to use the new set of very precise data (often better than for the Earth) in order to construct a model of the crust with a variable thickness and lateral density variations.

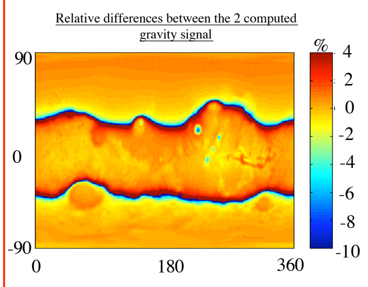
References :
Wieczorek, M. A., and R. J. Phillips, Potential anomalies on a sphere: Applications to the thickness of the lunar crust, *J. Geophys. Res.*, **103**, pp. 1715-1724, 1998.
McGovern, P. J., et al. Localized gravity/topography admittance and correlation spectra on Mars:Implications for regional and global evolution, *J. Geophys. Res.*, , pp. 1-18, 2002.
McKenzie, Dan. et al.,The relationship between Martian gravity and topography, *Earth and plan. Sci. Lett*., **195**, pp. 1-16, 2002.

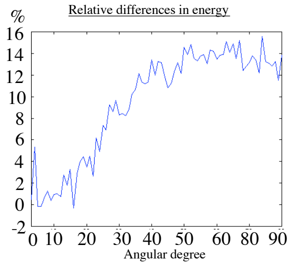
The mass-sheet approximation is not valid for high topography :

On Mars, in regions of high topography such as the Tharsis rise and its associated montes, the finite amplitude of topography has to be taken into account in order to obtain a good estimation of the gravity.

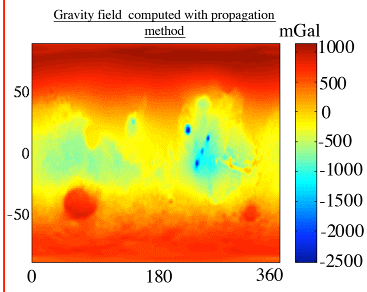
Gravity signal associated with the topography computed with the propagation method and with the mass-sheet approximation.

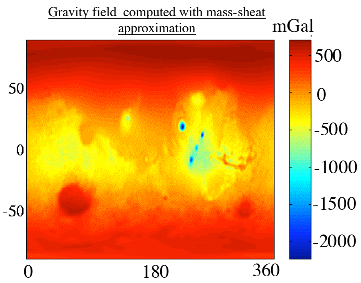
1. At spacecraft altitude (100 km)





2. Propagated to the surface





➘ The mass-sheet approximation tends to underestimate the gravity field for region of high topography.
➘ The errors are greater at the surface (with a mean difference of 35%).

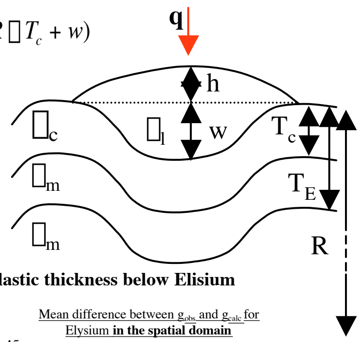
Application :Determination of elastic thickness

From the potential value on each density interface, the deflection *w* associated with a surface load can be determined from.

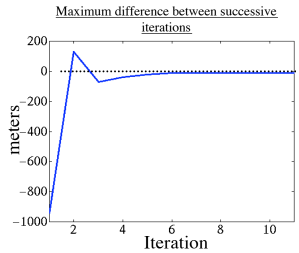
$$D\nabla^6 w + 4D\nabla^4 w + ET_E R^2\nabla^2 w + 2ET_E R^2 w = R^4(\nabla^2 + 1 - \nu)q$$

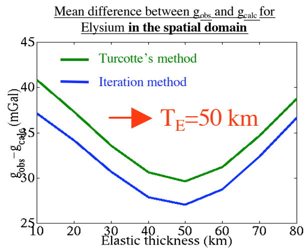
With
$$q = -\rho_c U(R+w) - (\rho_m - \rho_c)U(R - T_c + w)$$
(when $\rho_L = \rho_c$ and $T_E \leq T_c$)

⇒ in order to calculate q we need to know the potential on each density interface which depends on the flexure.
⇒ the above two equations need to be solved iteratively
⇒ convergence after ≈ 6 iterations



-**First application : Determination of the elastic thickness below Elisium**





- Consistent with the 50 km ≤ T_E ≤ 80 km of *McGovern et al 2002* (performed in spectral domain).
Inconsistent with T_E=25 km of *McKenzie et al. 2002* (in spectral domain).
- Differences between observed and calculated gravity are most likely due to subsurface loads.

-**Next step :**
- Include subsurface loads as well as surface loads with densities different from the crust.
- Determine geometry and magnitude of subsurface loads.
- Convert T_E estimates into heat flux estimates at the time of loading.